1. Prove that is not a regular language.

Suppose on the contrary that is regular, then by pumping lemma there exists some constant such that if , then there exist such that , and for any . In particular, take , then , and for any . As , we conclude for some . Hence, , contradicting the fact that .

1. For arbitrary constant , is regular or not?

No.

1. The decimal notation for a number is the number written in the usual way, as a string over the alphabet . For example, the decimal notation for 13 is a string of length 2. In unary notation, only the symbol “I” is used; thus 5 would be represented asIIIII in unary notation. Show that each of the following is or is not a regular language.

(For regular languages, write down its regular expression or describe the automata accepting it; for languages that are not regular, prove it using pumping lemma)

* 1. { is the unary notation for a number that is a multiple of 7}
  2. { is the unary notation for }

No. Suppose on the contrary that is regular, then by pumping lemma there exists some constant such that if and , then there exist such that , and for any . In particular, take , then , and for any . We know consists of , so let for some . Use the fact that , we know for every for some . This is impossible, as this implies that where . However, , the latter can be divided by while the former cannot, so they cannot be equal.